

ORIGINAL ARTICLE



A Link Joint Model for Vibration Isolation of Multi Parts Towed Cable System

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Abstract:

A linear equality relations from Gauss Principle to build model of link-joint system is adopted to simulate a multi parts towed cable system. The multi part design has a vibration isolation effect for three part and more. Length of main cable shows elastic effect for duration of oscillation. Tow speed of cable. A 2ed order linear system with different damping ratio is found in full turns. The heave oscillation is damped by multi part structure. It is an energy dissipation with high moment transportation of each part.

Keywords: Gauss Principle; vibration isolation; energy dissipation; multi parts towed cable system

Introduction

Two part towed cable system has a significant effect on the damp of the disturbance from a relative small pitch or sway of mothership. Wu[1] and Chwang[2] carried out . In a straight tow, two part cable-body system also shows a linear response to surface disturbance source. Amplitude attenuation but frequency consistency of vibration are observed from sea trails. However mothership speed loss and heading instability both lead slack of cable. The cable dropped and pulled up with shock wave transfers also lead an uncontrollable vibration of second part towed vehicle. However, turning maneuvers of multi cable-body system is a common searching trajectory near sea bed tow.

Several dynamic model can be adopted in simulation of towed cable system. The methods include a finite difference box scheme such as Albow [3], Wu [4], Wang [5], finite element method Sun [6], lumped parameters method Huang [7], Zhu [8], Ding[9]. In this design the lumped parameters scheme is more suitable for connection condition for each cable. Also the finite element modelling is applied in simulation

of towed cables, however, the important damping modeling is slightly inappropriate in a significant damping system.

Another link joint system modelling of lumped mass method based on Gauss Principle based on Skop[10] is verified and extensively applied in simulation of this type of towed cable system. The maneuver parameters from Chapman [11], Grosenbaugh [12] such as r/L and σ is fully simulated for a guidance to more efficient tow in marine geological tow exploration planning.

1 linear equality relations from Gauss Principle

A Cartesian coordinate system includes a series of mass particles m_1, m_2, \dots, m_n , Positions of the j^{th} particles is indicated as $s_j = (x_j, y_j, z_j)^T$ in this discrete system. The j^{th} particle is subjected to a given impressed force $f_j(t)$. This force is decomposed into three mutually perpendicular coordinate directions. The acceleration of this particle can be decided as a vector $a_j = F_j(t)/m_j$. The acceleration components of j^{th} particle can be the same as $F_j(t)$. Kinematics equation of each particle can be given as

$$m \cdot a(t) = f(\dot{x}(t), x(t), t)$$

Where

$$\begin{aligned} f(t) &= (f_1^T, f_2^T, \dots, f_n^T)^T \\ a(t) &= (a_1^T, a_2^T, \dots, a_n^T)^T \\ s(t) &= (s_1^T, s_2^T, \dots, s_n^T)^T \\ M &= \text{Diag}(m_1, m_1, m_1, \dots, m_n, m_n, m_n) \end{aligned}$$

The virtual mass are considered as $m_j = m_{0j} + m_{aj}$ for each element in mass matrix M.

Consider a constrained system subject to internal and external forces, where the constraint forces do no net work. Describing the system in generalized

coordinates, let $a(t)$ be the generalized accelerations of the unconstrained system (with non-constraint forces only). Suppose the system is in state $\{x, \dot{x}, \ddot{x}\}$ of generalized positions and velocities. Then, the true generalized accelerations of the constrained system are minimized as

$$R(x) = (\ddot{x} - a)^T M (\ddot{x} - a) = (M^{1/2} \ddot{x} - M^{1/2} a)^T (M^{1/2} \ddot{x} - M^{1/2} a)$$

The constraints M can be expressed as linear equality relations between the accelerations of particles in the system, the constraints can be given as a linear form

$$A(x) \ddot{x} = b(x, \dot{x}, t)$$

By using unique Moore-Penrose inverse, the minimization of $R(x)$ under constraint of this linear system. The acceleration of n particles system is given by

$$\ddot{x} = a + M^{-1/2} (AM^{-1/2})^+ (b - Aa)$$

In which $^+$ indicates unique Moore-Penrose inverse.

2 Model of link-joint system

Fig.2 has a system of links and joints with mothership, vibration isolation module and towed body. The cable is divided into N sections with buoys and control units as joints separation. Each

section contains one rigid link and two joints at both ends. The joint can be the same as mass particle. Forces acting on sections are lumped and concentrated at joints. The towed body is the bottom end of a cable and noted as s_N . The mothership is linked as the top end of a cable and noted as s_m . The linear equality relations can be rewritten as

$$A(x) \ddot{x} = b(\dot{x}, \dot{s}_m, \ddot{s}_m)$$

In which

$$A = \begin{bmatrix} (s_1 - s_m)^T & 0 & \dots & 0 \\ -(s_2 - s_1)^T & (s_2 - s_1)^T & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & -(s_N - s_{N-1})^T & (s_N - s_{N-1})^T \end{bmatrix}$$

$$b = -[\|\dot{s}_1 - \dot{s}_m\|^2 + (s_1 - s_m)^T \ddot{s}_m, \|\dot{s}_2 - \dot{s}_1\|^2, \dots, \|\dot{s}_N - \dot{s}_{N-1}\|^2]^T$$

According to Gauss principle, solution of the linear system under the constraint (4) is the acceleration of cable joints. The constrained joints system is

$$u_1 = \|s_1 - s_m\|^2 = l^2, u_i = \|s_{i+1} - s_i\|^2 = l^2, \dots, u_N = \|s_N - s_{N-1}\|^2 = l^2$$

where $l=L/N$, L is the cable length. These positions constraints could form a matrix as

$$\theta(u) = [u_1, \dots, u_N]^T - [l^2, \dots, l^2]_{1 \times N}^T$$

The time differential of $\theta(u)$ gives a velocity

$$\vartheta(u) = \frac{\partial \theta}{\partial t} = [(s_1 - s_m)^T (\dot{s}_1 - \dot{s}_m), (s_2 - s_1)^T (\dot{s}_2 - \dot{s}_1), \dots, (s_N - s_{N-1})^T (\dot{s}_N - \dot{s}_{N-1})]^T$$

A constrained system is modified as

$$\ddot{x} = a + M^{-1/2}(AM^{-1/2})^+(b - Aa) - \alpha \left(\frac{\partial \theta}{\partial x}\right)^T \theta - \beta \left(\frac{\partial \vartheta}{\partial x}\right)^T \vartheta$$

where the control coefficient α, β are used to keep numerical stability of linear system (4), and to simulate damp effect of structure in a conservative system.

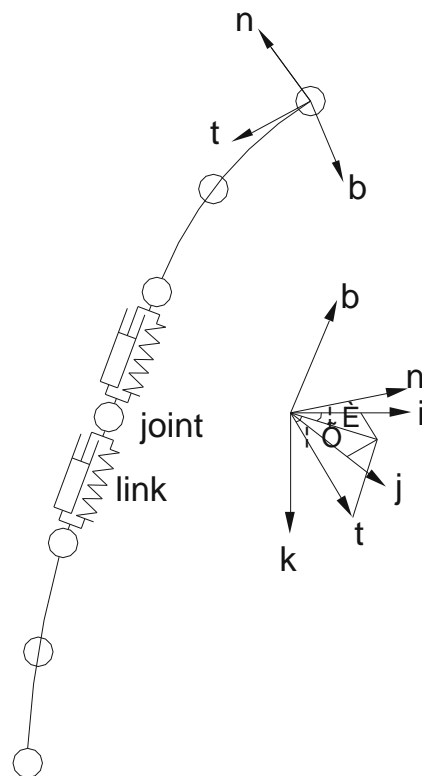


Figure1 link-joint system to model towed cable in local and global system

The towed body is modeled by 3 degrees of freedom model rather than a 6 DOF system. It can be treated as a discrete joint so that the cable-body part is more conveniently implemented. This simplified model has a better numerical efficiency and stability with in a multi cable-system.

Forces acting on the joints includes link tension f_0 , hydrodynamics force f_j , gravity and buoy force f_2 . These force can be transformed by a coordinate matrix into global linear system. The acceleration on j^{th} joint due to external force is

$$a_j = f_{0j} / m_j + f_{1j} / m_j + f_{2j} / m_j$$

Tension F_0 is simulated by axis of link balance of link forces acting on joint.

$$F_{0r} = ES \left(\frac{L_j^{i+1}}{L_j^i} - 1 \right)$$

where L_j is length of link between j and $j+1$. Superscript i denotes the length at i^{th} time step. A transform matrix from local coordinates to global system $(i, j, k)f_0=(F_{0r}, 0, 0)[R]$

$$R = \begin{bmatrix} \cos \theta \cos \phi & -\cos \theta \sin \phi & \sin \theta \\ \sin \theta \cos \phi & -\sin \theta \sin \phi & -\cos \theta \\ \sin \phi & \cos \phi & 0 \end{bmatrix}$$

The hydrodynamics force can be simulated as function of attack angles and frequency. The force on each links

$$F_{1t} = \frac{1}{2} \rho C_{dt} V_t^2 D l, \quad F_{1n} = \frac{1}{2} \rho C_{dn} V_n^2 D l, \quad F_{1b} = \frac{1}{2} \rho C_{db} V_b^2 D l$$

A transform matrix from local coordinates to global system $f_1(i, j, k)=(F_{1t}, F_{1n}, F_{1b})[R]$ is used in transformation.

The incident angle α of links are in range of $[0^\circ,$

$90^\circ]$ for coming flow. The drag coefficients of each link can be obtained by interpolation of experimental data with α . The expressions at local coordinates are such as

$$\begin{cases} 0^\circ < \alpha < 30^\circ \\ 30^\circ < \alpha < 90^\circ \end{cases} \begin{cases} C_{Dt} = D_0(0.273 + 0.827\alpha^2) \cos \alpha \\ C_{Dn} = D_0(0.273 + 0.827\alpha^2) \sin \alpha \\ C_{Dt} = D_0 \sin \alpha \cos \alpha \\ C_{Dn} = D_0 \sin^2 \alpha \end{cases}$$

In which D_0 is the flow drag coefficients curve of cylinder according to Fig.1.

The vortex shedding in wake of each link leads to

unstable motion of vibration of cable in a high frequency. The drag coefficients is given as

$$D_0(t) = C_D(\alpha) \left[1 + 10 \left(\frac{d^2}{m_c} \right)^2 \right] \sin(2\pi f_c t + \psi_0)$$

In which f_c is the frequency of shedding frequency, m_c is the added mass of cable link. Ψ_0 is the phase angle.

$$f_v = \frac{St |\mathbf{V}|}{d} \sin \alpha$$

St is Strouhal number of a cylinder flow. The range of St is shown in Fig.2. It indicates a small step response characteristic in Reynolds number from 3.6×10^5 to 3.5×10^6 leads to a larger vibration.

In a range of towed Reynolds number the St number is considered as constant. The flow induced strum is a low frequency excitation.

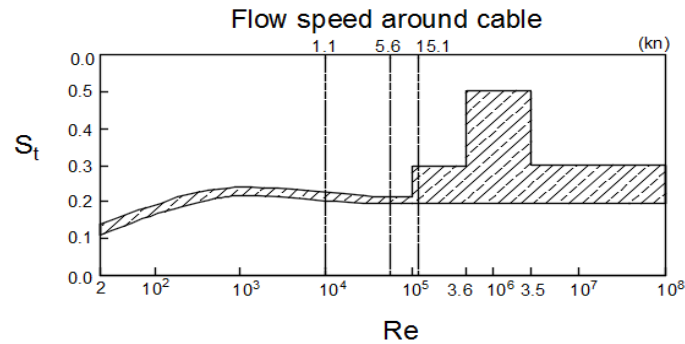


Figure 2 The Strouhal number and tow speed range around cable

3 Multi-segments towed cable system maneuver

3.1 Multi-segments towed system

In a multi segment tow the heave and yaw of mothership caused by high sea states is mostly damped by depressor and several segments of low tensional cables with damped units. The towed aquatic sonar is attached at end of cable. The turning manipulation in a searching seabed area can be high efficiently and precisely carried out

with a vibration isolation towed system.

The mothership turning maneuver includes full turn and U turn. For different tow speed the response of towed system has damped oscillation and resonance. Long period decay of energy conversion can be switched by damp unit as shown in Fig.3. This unit has hydrodynamic damping from a truss-framed with significant added mass effect.

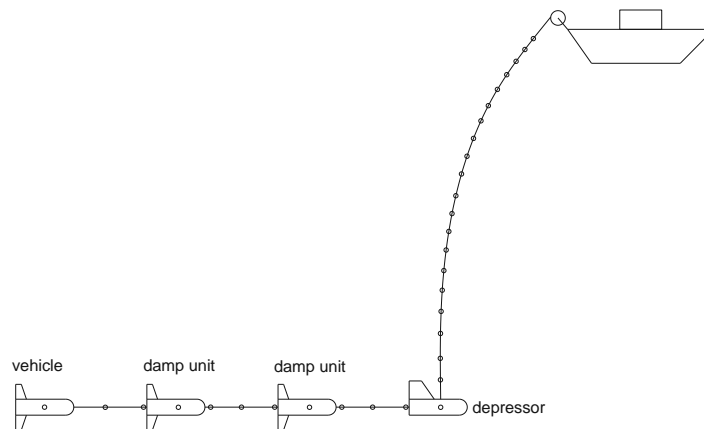


Figure 3 A multi parts towed cable system

The multi segments cable-body system has one to three parts of 200m low tensional cable-body. The added mass of damp unit reach its designed maximum for a damp efficiency. When the disturbance energy from mothership and high tensioned cable is restrained or absorbed via a damp unit from attached water, the energy downstream may be minimized. In a full turn the cable has a twisting and bending behavior, this may lead a structural damage for cable connections at depressor and damp unit. Swivel joint and flexible electrical connection is usually

adapted in this towed system. In area of searching sea bed, U turn and full turn manipulations for tow programming have to formulate a scheme to decide turning radius and extending scanning coverage.

3.2 Vibration isolation in damped oscillation in turning maneuver

A design of tow multi part cable system is given for maneuvers. The main cable length is 2200m with weight of 0.750kg/m and diameter of 0.017m. The depressor has a net weight of 700 kg

with the projected area in local coordinate system (t, n, b) are 0.5m^3 and drag coefficients of 1.2. The low tensional cable of following parts has a weight of 0.725kg/m and diameter of 0.030m . Each towed vehicle has a mass of 135kg but a net weight near zero in seawater and projected area in local coordinate system (t, n, b) are 1.0m^3 and drag coefficients of 1.2. The towed vehicle has a significant added mass of 135kg .

It is an evaluation of a dynamic response of cable-body maneuvers with different structure parameters. A turning loop is simulated for these designs. The turning ability of ship decides a tow condition of cable-body system. In this simulation the full turn radius includes 250m , 500m , 750m , 1000m , etc. Main cable has a length of 1000m to 3200m for a large depth searching. In a full turn the turning speed from 1 to 6 knot are simulated. Snap load on main cables are significant in a multi towed system as shown in Fig.4.

Horizontal trajectories are given in Fig.4. For a large turning radius and main cable length ratio R/L from 0.114 to 0.682 the strum is amplified to a periodical oscillation in a large ratio full turn. In a small turn, the horizontal swing for the low tensional parts is not observed. This means a torsional cable connector of depressor, although the bending and twisting behavior are not consider in this model.

Heaving history of single part, two part, three part in full turn are shown in Fig.5. It is evident that the third part has a stable falling down history. The second part plays a role in vibration attenuation from a full turn disturbance. In a large R/L the turning shows a better performance in heave motion. However, the covering distance is usually too far to measuring limits for a towed vehicle.

Tension history gives the same pattern as heave motion in simulation of full turns as shown in

Fig.6. Snap load in cables exhibits multiple impacts, that is, fatigue may caused by a fatigue load of cycling form. For a mechanical joint it is necessary to start overhaul. For a fixed depth drag, it is necessary to carry out a detection of defects in deck hanging point of cable segment in practice.

At $R/L = 0.144$ heave period is 8 minutes 43second. The hydrodynamic damp is so small that a persistence can be found in these simulations. These follow a 2ed order linear system of damping ratios increasing with towing speed. For a small ratio heave motion has a relative small amplitude attenuation.

There are lengths of 1000m to 3200m with 200m . the parameter $\sigma = ml/M$ is from 1.071 to 3.429. M is total mass of each towed body. A long cable shows a behavior similar to oscillating spring for turning disturbance. This mass distribution property of a towed system shows the resonance is in proportion to the parameter σ as shown in Fig.7. The heave period is in reverse ratio to σ . The damp effort also decrease with σ . The σ also indicates an effort of elastic vibration. The low tensional parts suppress the elastic vibration in turnings.

Fig.8 to Fig.11 are the heave history and horizontal trajectory of two part towed system in U turns, all heave motions show a periodic heave decreases with the turning radii and turning speed. The heave history of $R=250\text{m}$ is validated by sea trail data in Fig.9. In $R=500\text{m}$, there is only one heave impact on the main cable, however for a small tow the sharp falling down induces a significantly small damp for a long periodic heave motion. Also the falling down and recover of low tensional parts are also a slow motion. In comparison of 2knot, 4knot, 6knot the averaged heaving period is becoming large. This gives a better stability in U turn.

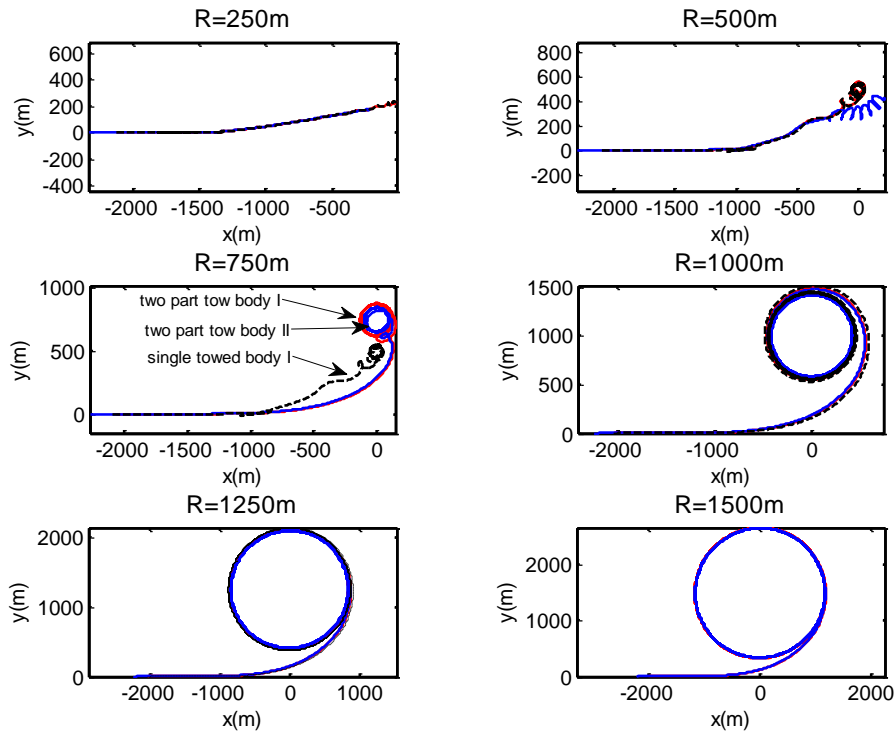


Figure 4 Horizontal trajectory of towed body in full turns of multi part towed cable system of different turning radius.

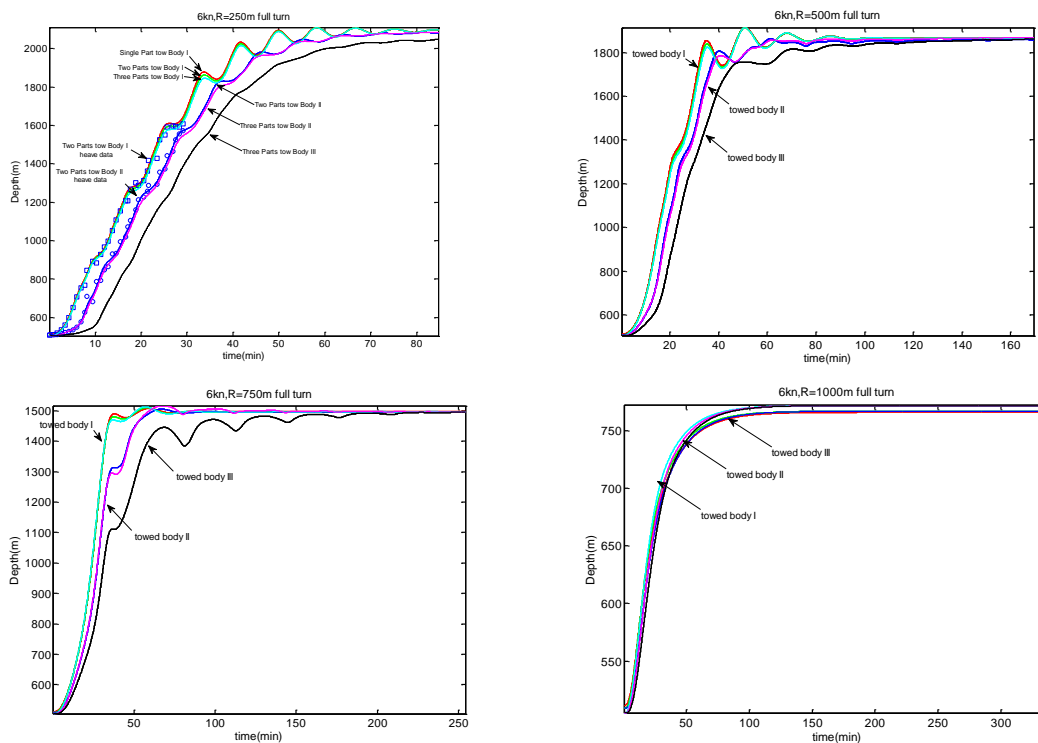


Figure 5 Heave history of depressor (body I), damp unit (body II) and towed body (body III) at cable free end, main cable length 2200m, at tow speed 6 knot

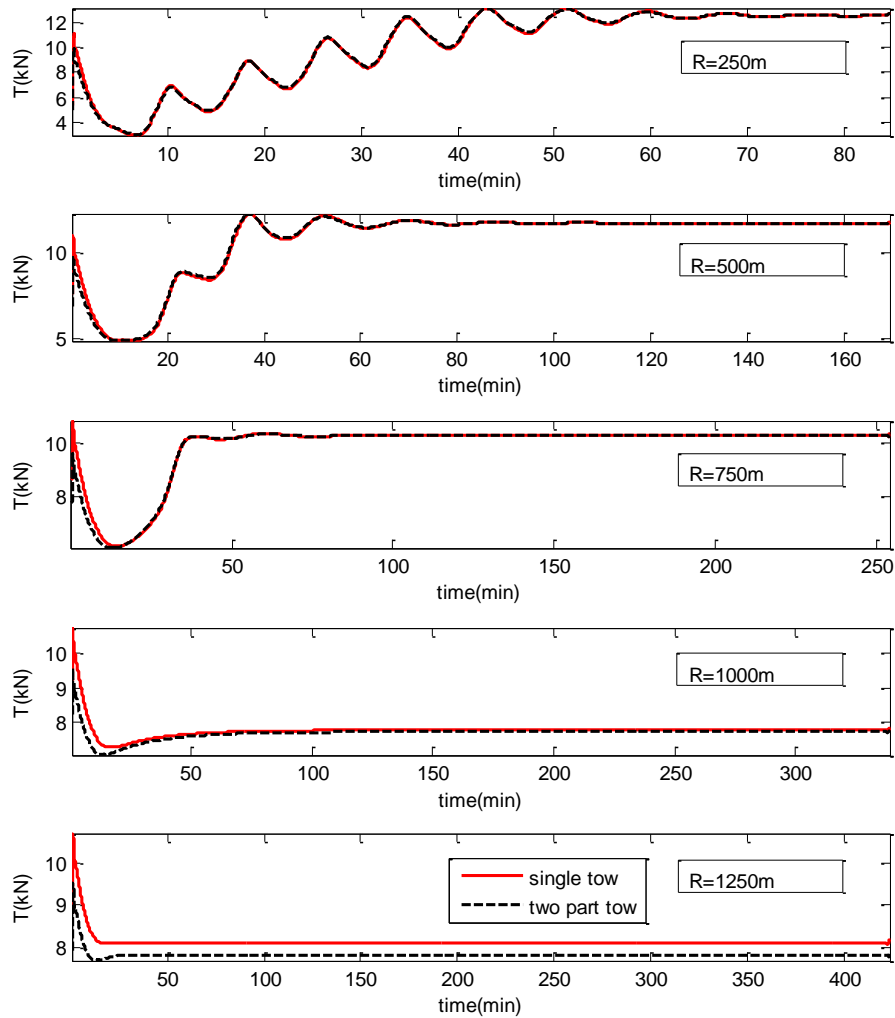
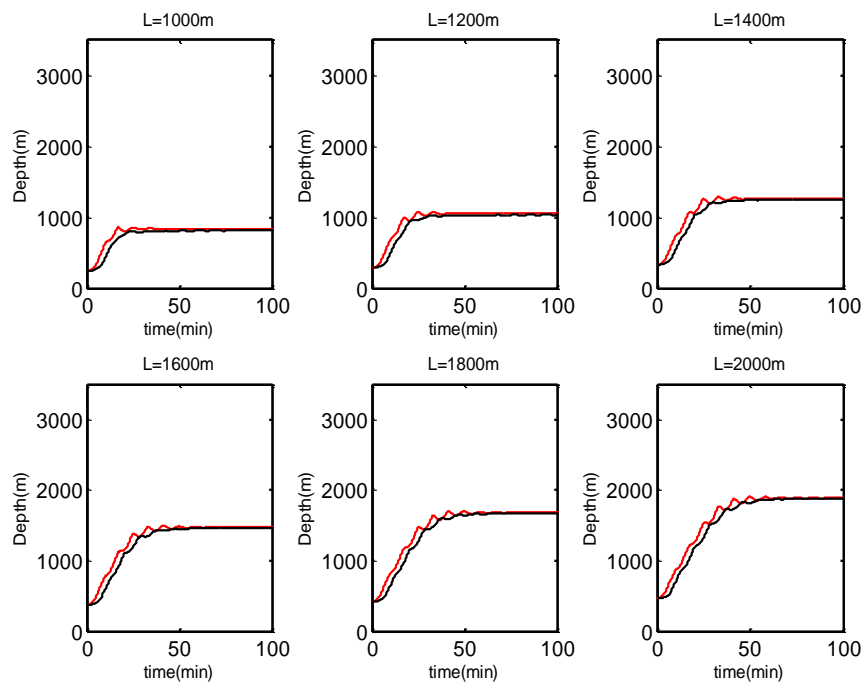


Figure 6 Tension history of main cable, of a single tow and two part tow, main cable length 2200m, at tow speed 6 knot



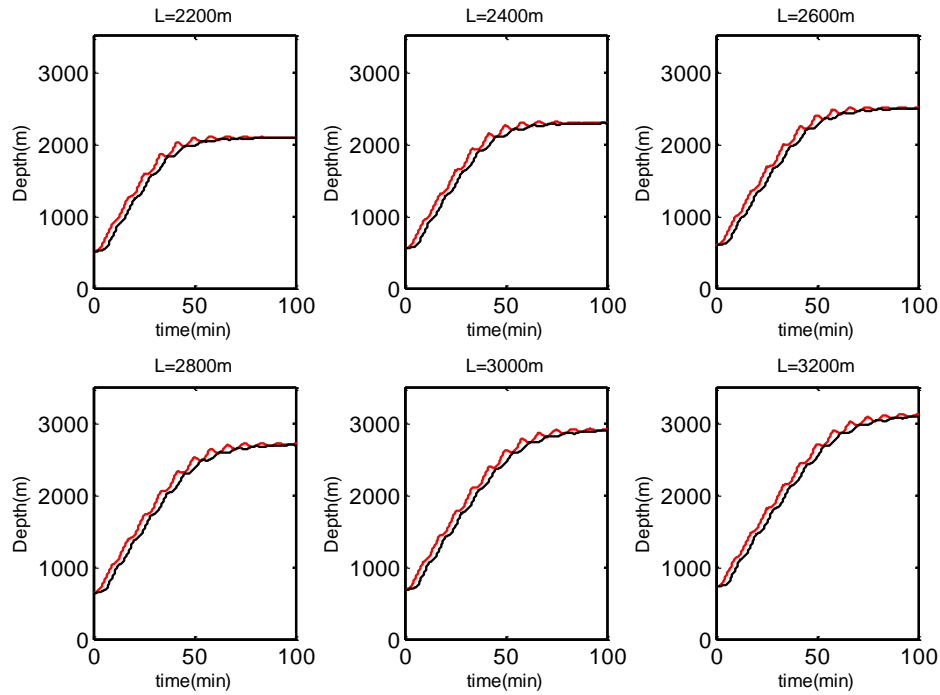


Figure 7 Heave history of depressor (red line) and towed body(black line) of different main cable length at towed speed 6kn of R=250m in full turns

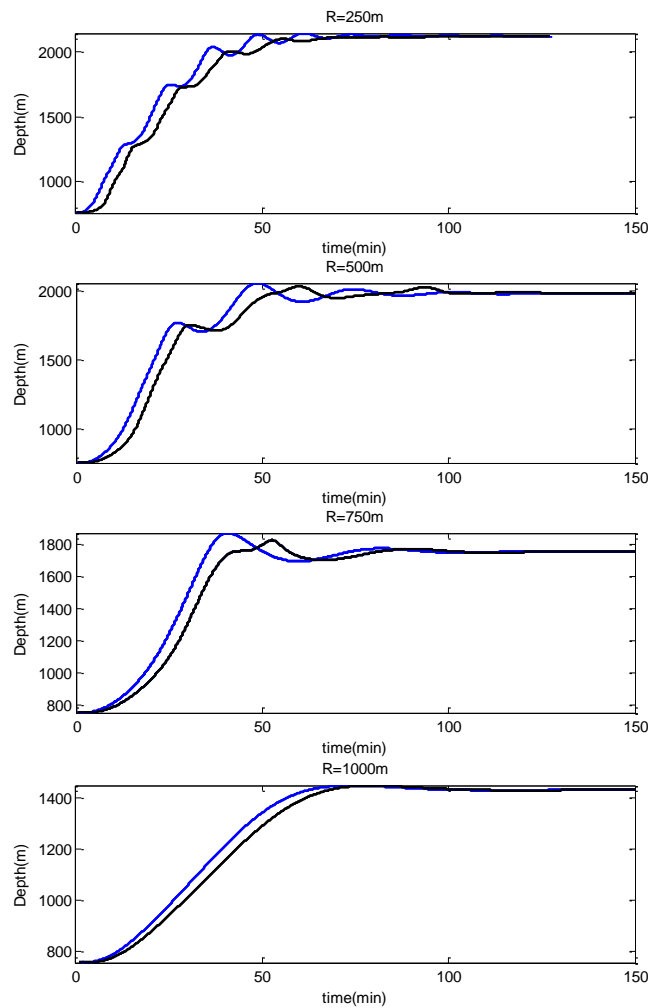


Figure 8 Heave history of depressor(body I) and towed vehicle(body II) at cable free end, main cable length 2200m, at tow speed 2 knot, in U turns.

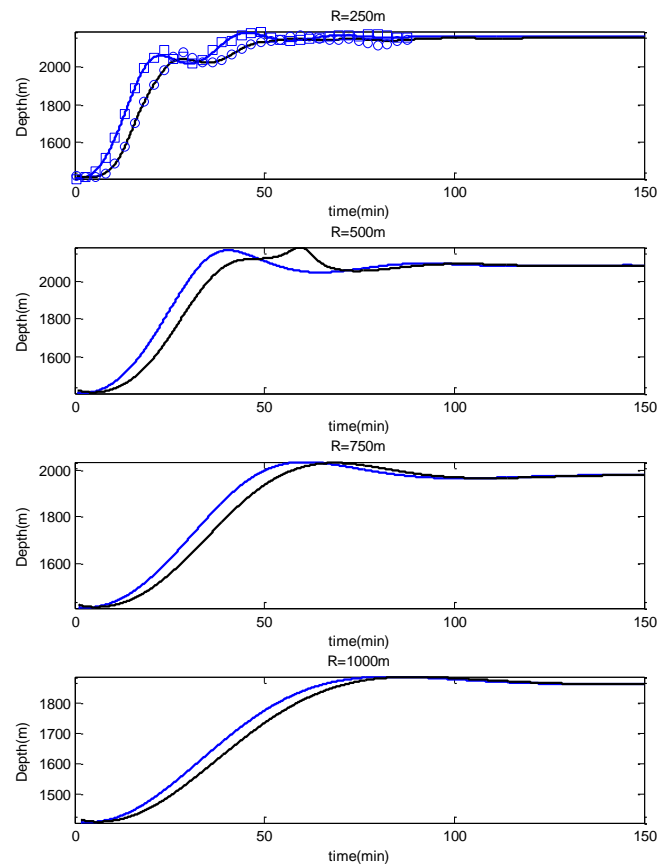


Figure 9 Heave history of depressor (body I), towed body (body II) at cable free end, main cable length 2200m, at tow speed 4 knot, in U turns and compared with sea trail data

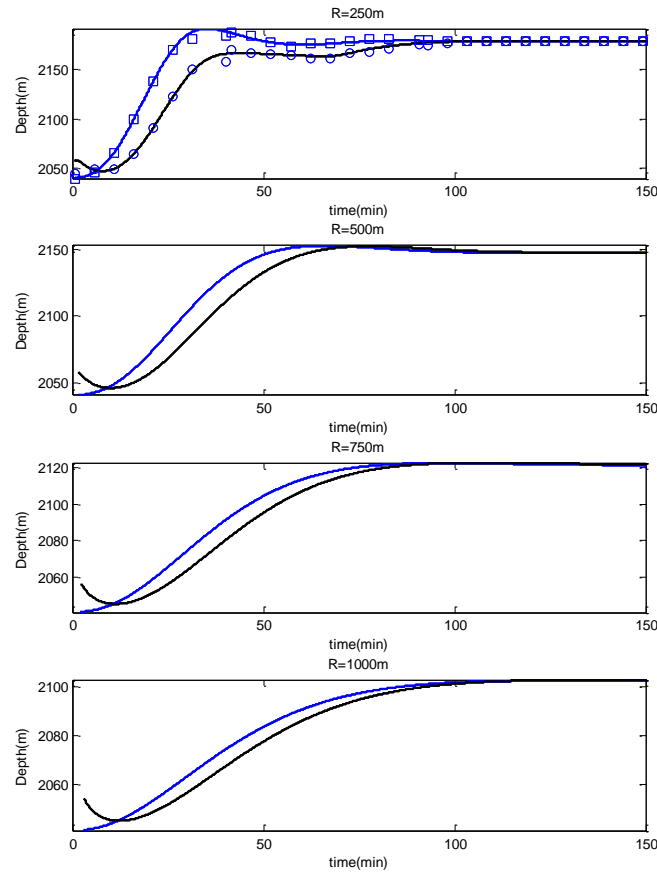


Figure 10 Heave history of depressor (blue line) and towed body (black line) of different main cable length at tow speed 6kn, in U turns

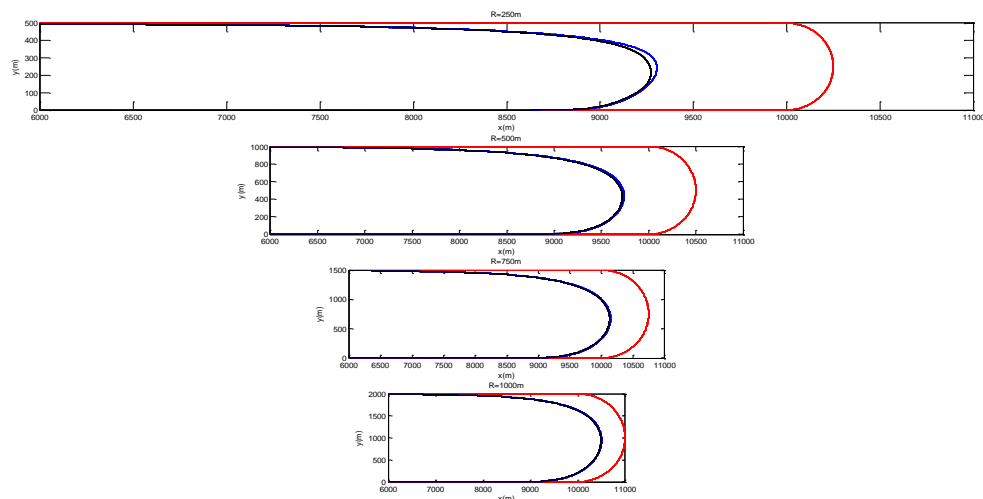


Figure 11 Horizontal trajectory of towed body in U turns of multi part towed cable system of different turning radius.

Conclusion

A linear dynamic system is built to simulate multi part towed cable system. The maneuver hydrodynamic parameters r/L are given for a full inspection of dynamic response for multi part tow. There are longer periodical heave motion of depressor for a full turn than the surface wave disturbance. In three part tow and more, the heave motion is totally damped by the adjacent part. In full turns a 2ed linear system is found in heave history. Snap in cables is significant for a small r/L . Duration of oscillation of depressor is proportional to the mass maneuver mass parameters σ .

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