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Evolutionary Game and Simulation Analysis of Community Group-Buying Platforms Supervision

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Abstract: To improve the efficiency of the community group-buying platforms supervision and promote the sustainable and healthy development of the community group-buying platforms, this paper establishes a three-party game model of the community group-buying platforms, suppliers and consumers based on evolutionary game theory, determines the conditions for the three parties to reach the ideal state (Positive supervision,Honest transactions,Protecting rights). Finally, using MATLAB software to conduct numerical simulation to verify the validity of evolutionary stability analysis and analyze the factors that affect the rate of suppliers choosing "Honest transactions", and puts forward suggestions to improve the efficiency of the community group-buying platforms.

Keywords: Community group-buying platforms; Supervision efficiency; Three-party game; Numerical simulation

1. Introduction

In recent years, influenced by the new crown pneumonia epidemic, community group-buying platforms such as Meituan Selected and DuoDuo buy vegetables have come online, which are based on communities and serve communities, such as social networks, commodity retail, life services and other entrance scenarios, establishing communities online and paying for them, with the platforms providing supply chains for offline services[1] . It is popular among consumers because it delivers food to their doorstep without leaving their homes and the prices are unbelievably cheap. However, the consumers' experience is often affected by problems in the transaction, such as the quality of some suppliers' products not being up to par, price gouging, non-delivery after payment and incomplete information about the goods.

At the same time, due to the cheap price of the products, most consumers do not bother to pursue responsibility and thus their consumer rights are not protected. The platforms also take the supervision of suppliers lightly due to the low rate of consumers complaints.

In December 2020, the State Administration of Market supervision, in conjunction with the Ministry of Commerce, organized an administrative guidance meeting on regulating the order of community group-buying, with the participation of six Internet platform companies, including Alibaba, Tencent, Jingdong, Meituan, Jindo and DDT.

The meeting called for strengthening the supervision of community group-buying price practices and anti-unfair competition in accordance with the law, regulating the order of the community group-buying market, maintaining a fair competitive market environment, and ensuring that people's livelihoods are effectively protected and improved [2].

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Most scholars currently focus on the factors influencing consumers usage behaviour in community group purchasing. Zeng Jing et al[3] , by investigating and analyzing customers' use of the platforms in the community group-buying model, obtained that the quality of products in community group-buying has more direct influence on consumers' use than the shopping environment; Yuan Yuxin et al [4, by studying the factors and degree of influence on residents' willingness to use group-buying platforms in Baotou, obtained that the quality of platform services has the greatest influence on residents' willingness to use; some scholars are also concerned about the development status and future trend of group purchasing. Wang Ruilian and Huang Kaijin [5] analyzed the current situation and the strengths and weaknesses of community group-buying and proposed corresponding strategies to provide reference for the development of community group-buying; on the basis of the definition of community group buying industry, Xiao Jianli[6]analyzed the characteristics of the development of community group buying, pointed out that community group buying has deficiencies such as price war and difficult effective;Ying Yike and Cui Tui et al [7] studied the satisfaction and development of community group buying by using factor analysis and multiple response analysis, and found that people expect further improvement in the quality, on-time delivery and product information of community group buying; Liu Xiaochun et al [8]suggested that the supervision of community group buying should focus on food safety and consumers rights protection.

Since there is a lack of research results related to the evolutionary game analysis of community group-buying platforms, this paper establishes a three-party game model from the perspective of evolutionary game to explore the initial conditions for community group-buying platforms, suppliers and consumers to reach the ideal state, and makes suggestions for improving the efficiency of platforms supervision.

2. Model assumptions and establishment

The following assumptions are made based on the actual situation of the community group-buying platforms, suppliers and consumers.

1.The community group-buying platforms, suppliers and consumers participants are all finitely rational. The strategy choice space of community group-buying platforms is {Positive supervision, Negative supervision}; the strategy choice space of suppliers is {Honest transaction, Fraudulent transaction}; the strategy choice space of consumers is {Protecting rights, No protecting rights}.

2.The probability of positive supervision of community group-buying platforms is x, and the probability of negative supervision is $1-x$; the probability of suppliers' honest transaction is y, and the probability of suppliers' fraudulent transaction is $1-y$; the probability of consumers protecting their rights is z, and the probability of consumers not protecting their rights is $1-z$.

3.The basic benefit of community group-buying platforms supervision is Q, the cost of positive supervision of suppliers by community group-buying platforms is F ,and the cost of negative supervision of suppliers is negligible.In the case of consumers protect their rights, positive supervision of community group-buying platforms will gain the trust of consumers, recorded as B_1, community group-buying platforms negative supervision and failure to find suppliers' fraudulent transactions will cause consumers dissatisfaction, recorded as B_2.

4. The price of the suppliers' sales product is R, the cost of honest transactions is C_1, the cost of

fraudulent transactions is C_2, recorded ΔC as $C_1 - C_2$. When suppliers deceive transactions, if they are found by the platforms, they will be fined P, if consumers protect their rights, suppliers will lose own reputation, which will be recorded as B_3.

5.When the suppliers trade honestly,the utility obtained by the consumers is K_1;When suppliers engage in fraudulent transactions, the utility obtained by the consumers is K_2 $(K_1 > K_2)$, the cost of protecting the rights of consumers is C_0, when the consumers protect their rights, they will be subsidized by the community group-buying platforms, recorded as A, and when the community group-buying platforms find that the suppliers do have fraudulent transactions, it will also reward the consumers, recorded as V.

6.When suppliers conduct fraudulent transactions, as long as community group-buying platforms choose to positively supervise, it will be found. However, if community group-buying platforms carry out negative supervision, suppliers' fraudulent transactions may or may not be detected. The probability that the community group-buying platforms can detect the suppliers' fraudulent transactions is θ, and the probability that the suppliers' fraudulent transactions are not found is $1\text{-}\theta$.

Based on the assumptions of the above model, this paper constructs a three-party strategy game matrix involving the community group-buying platforms, suppliers and consumers, as shown in Table 1.

Table 1 Three-party payoff matrix

Strategy choice			Community group-buying platforms: Positive supervision（x）	Community group-buying platforms: Negative supervision（$1-x$）
Suppliers	Honest transaction（y）	Consumers protect their rights (z)	$R-C_1$ K_1-C_0+A $Q-F+B_1$	$R-C_1$ K_1-C_0+A Q
		Consumers do not protect their rights（$1-z$）	$R-C_1$ K_1 $Q-F$	$R-C_1$ K_1 Q
	Fraudulent transaction（$1-y$）	Consumers protect their rights (z)	$R-C_2-P-B_3$ K_2-C_0+A+V $Q-F+B_1+P$	$R-C_2-\theta(P+B_3)$ $K_2-C_0+A+\theta V$ $Q+\theta P-(1-\theta)B_2$
		Consumers do not protect their rights（$1-z$）	$R-C_2-P$ K_2 $Q-F+P$	$R-C_2-\theta P$ 0 $Q+\theta P$

3. Equilibrium analysis of the evolution of community group-buying platforms, suppliers and consumers

3.1 Analysis of the replication dynamics of the community group-buying platforms

Let E_{p1} the expected benefits of choosing "Positive supervision" for community group-buying platforms, E_{p2} is the expected benefits of choosing "Negative supervision" .The average expected return for community group-buying platforms is $\overline{E_p}$, then we have:

$$\begin{aligned} E_{p1} &= yz(Q-F+B_1)+y(1-z)(Q-F) \\ &+(1-y)z(Q-F+B_1+P)+(1-y)(1-z)(Q-F+P) \\ &= Q-F+B_1z+P(1-y) \end{aligned} \quad (1)$$

$$\begin{aligned} E_{p2} &= yzQ+y(1-z)Q+(1-y)z[Q+\theta P-(1-\theta)B_2] \\ &+(1-y)(1-z)(Q+\theta P) \\ &= Q+(1-y)\theta P-(1-y)(1-\theta)zB_2 \end{aligned} \quad (2)$$

$$\overline{E_p} = xE_{p1}+(1-x)E_{p2} \quad (3)$$

The replication dynamic equation for the community group-buying platforms is

$$\begin{aligned} F(x) &= \frac{dx}{dt} = x(E_{c1}-\overline{E_c}) \\ &= x(1-x)[B_1z-F+(1-\theta)P(1-y)+B_2(1-y)(1-\theta)z] \end{aligned} \quad (4)$$

The replication dynamic equation for the probability of choosing the "Positive supervision" strategy for the community group-buying platforms is derived by taking a partial derivative:

$$F'(x) = (1-2x)[B_1z-F+(1-\theta)P(1-y)+B_2(1-y)(1-\theta)z] \quad (5)$$

The fact that $F(x)\equiv 0$ holds when $z=\dfrac{F-(1-\theta)P(1-y)}{B_1+B_2(1-y)(1-\theta)}=z^*$ implies that all levels are in a steady state; the fact that $F'(x)|_{x=0}<0$ and $F'(x)|_{x=1}>0$ holds when $0<z<z^*<1$, $x=0$ is an evolutionary stable strategy, i.e, when the probability of consumers choosing the strategy of "Protecting rights" is less than z^*, the community group-buying platforms will eventually tend to choose the strategy of "Negative supervision";the fact that $F'(x)|_{x=0}>0$ and $F'(x)|_{x=1}<0$ holds when $0<z^*<z<1, x=1$ is an evolutionary stable strategy, i.e when the probability of consumers choosing the strategy of "Protecting rights" is greater than z^*, community group-buying platforms will eventually tend to choose the strategy of "Positive supervision".

3.2 Supplier replication dynamic equation analysis

Let the expected return of suppliers choosing "Honest transaction" be E_{g1}, the expected return of suppliers

choosing "Fraudulent transaction" be E_{g2}, and the average expected return of suppliers be $\overline{E_g}$. Then we have:

$$\begin{aligned} E_{g1} &= (R-C_1)xz+(R-C_1)(1-x)z+(R-C_1)x(1-z) \\ &+(R-C_1)(1-x)(1-z) \\ &= R-C_1 \end{aligned} \tag{6}$$

$$\begin{aligned} E_{g2} &= (R-C_2-P-B_3)xz+[R-C_2-\theta(P+B_3)](1-x)z+ \\ &(R-C_2-P)x(1-z)+(R-C_2-aP)(1-x)(1-z) \\ &= (R-C_2-P)x+(R-C_2-\theta P)(1-x)+(1-\theta)B_3xz-\theta B_3 \end{aligned} \tag{7}$$

$$\overline{E_g} = yE_{g1}+(1-y)E_{g2} \tag{8}$$

The replication dynamic equation for suppliers is

$$\begin{aligned} F(y) &= \frac{dy}{dt} = y(E_{g1}-\overline{E_g}) \\ &= y(1-y)[(R-C_1-(R-C_2-P)x-(R-C_2-\theta P)(1-x)-(1-\theta)B_3xz+\theta B_3] \end{aligned} \tag{9}$$

The partial derivative of the replication dynamic equation for the probability that suppliers choose the "Honest transaction" strategy yields:

$$F'(y) = (1-2y)[(R-C_1-(R-C_2-P)x-(R-C_2-\theta P)(1-x)-(1-\theta)B_3xz+\theta B_3] \tag{10}$$

The fact that $F(y)\equiv 0$ holds when $x=\frac{C_1-C_2-P-\theta B_3}{(1-\theta)(P+B_3z)}=x^*$ implies that all levels are in a steady state,the fact that $F'(y)|_{y=0}<0$ and $F'(y)|_{y=1}>0$ holds when $0<x<x^*<1, y=0$ is an evolutionary stable strategy,i.e. when the probability of community group buying platforms choosing the "Positive supervision" strategy is less than x^*, suppliers will eventually tend to choose the "Fraudulent transaction" strategy. The fact that $F'(y)|_{y=0}>0$ and $F'(y)|_{y=1}<0$ holds when $0<x^*<x<1, y=1$is an evolutionary stable strategy,i.e when the probability of community group-buying platforms choosing the "Positive supervision" strategy is greater than x^*, suppliers will eventually tend to choose the "Honest transaction" strategy.

3.3 Dynamic equation analysis of consumers replication

Let the expected benefit of consumers choosing to "Protecting rights" be E_{c1}, the expected benefit of consumers choosing to "No protecting rights" is E_{c2}, and the average expected benefit of consumers is $\overline{E_c}$, then we have:

$$E_{c1} = xy(K_1 - C_0 + A) + (1-x)y(K_1 - C_0 + A) + x(1-y)(K_2 - C_0 + A + V) + (1-x)(1-y)(K_2 - C_0 + A + \theta V) = y(K_1 - C_0 + A) + (1-y)(K_2 - C_0 + A) + Vx(1-y) + \theta V(1-x)(1-y) \quad (11)$$

$$E_{c2} = K_1 xy + K_1(1-x)\, y + K_2 x(1-y) \quad (12)$$

$$\overline{E_c} = zE_{c1} + (1-z)E_{c2} \quad (13)$$

The replication dynamic equation for consumers is

$$F(z) = \frac{dz}{dt} = z\left(E_{c1} - \overline{E_c}\right) = z(1-z)\{K_2(1-y)(1-x) - C_0 + A + Vx(1-y) + \theta V(1-x)(1-y)\} \quad (14)$$

The partial derivative of the replication dynamic equation for the probability that consumers choose the "Protecting rights"strategy gives

$$F'(z) = (1-2z)\{K_2(1-y)(1-x) - C_0 + A + Vx(1-y) + \theta V(1-x)(1-y)\} \quad (15)$$

The fact that $F(z) \equiv 0$ holds when $y = 1 - \frac{C_0 - A}{Vx + K_2(1-x) + \theta V(1-x)} = y^*$ implies that all levels are in a steady state; the fact that $F'(z)|_{z=0} > 0$ and $F'(z)|_{z=1} < 0$ holds when $0 < y < y^*, z = 1$ is an evolutionary stable strategy, i.e when the probability of suppliers choosing the "Honest transaction" strategy is less than y^*, consumers will eventually tend to choose the "Protecting rights" strategy; the fact that $F'(z)|_{z=0} < 0$ and $F'(z)|_{z=1} > 0$ holds when $0 < y^* < y < 1,\ z = 0$ is the evolutionary stabilization strategy, i.e. when the probability of suppliers choosing the "Honest transaction" strategy is greater than y^*, the consumers will eventually tend to choose the "No protecting rights" strategy.

3.4 Three-party evolutionary strategy analysis

The equilibrium point of each participant in the replicated dynamic system can be obtained by linking them together. In a multi-group evolutionary game, the combination of evolutionary stable strategies for a replicating dynamic system can only be a pure strategy equilibrium[9] , so we only need to discuss

$E_1(0,0,0), E_2(1,0,0), E_3(1,1,0), E_4(1,1,1), E_5(0,1,1), E_6(0,0,1), E_7(0,1,0), E_8(1,0,1)$ the asymptotic stability of these eight equilibrium points.

According to the Friedeman Evolutionary Stable Strategy (ESS), the Jacobi matrix can be applied to the local stability analysis of the evolutionary strategy of a population dynamics described by a differential system[10] , resulting in the following Jacobi matrix J .

$$J=\begin{pmatrix} \frac{\partial F(x)}{\partial x} & \frac{\partial F(x)}{\partial y} & \frac{\partial F(x)}{\partial z} \\ \frac{\partial F(y)}{\partial x} & \frac{\partial F(y)}{\partial y} & \frac{\partial F(y)}{\partial z} \\ \frac{\partial F(z)}{\partial x} & \frac{\partial F(z)}{\partial y} & \frac{\partial F(z)}{\partial z} \end{pmatrix} \quad (16)$$

According to the relevant knowledge points of evolutionary game theory and Liapunov's stability discriminant method, we know that if any equilibrium point is substituted into the Jacobi matrix, if all the eigenvalues of this matrix are negative real parts, then it is known that this equilibrium point is asymptotically stable and the system will gradually be in a stable state; otherwise, the equilibrium point is unstable and the system will also be in an unstable state [11]. In the following we conduct stability analysis of the dynamic system equilibrium points, and the corresponding eigenvalues of each equilibrium point are shown in Table 2.

Table 2 Stability analysis of equilibrium points

Equilibrium points	Eigenvalues		
(0,0,0)	$-F+P(1-\theta)$	$C_2-C_1+\theta P+\theta B_3$	$K_2-C_0+A+\theta V$
(1,0,0)	$F-P(1-\theta)$	$C_2-C_1+\theta B_3+P$	$A-C_0$
(1,1,0)	F	$C_1-C_2-P-\theta B_3$	$A-C_0$
(1,1,1)	$F-B_1$	$C_1-C_2-P+B_3(1-2\theta)$	C_0-A
(0,1,1)	B_1-F	$C_1-C_2-\theta\ (P+B_3)$	C_0-A
(0,0,1)	$-F+B_1+(B_2+P)(1-\theta)$	$C_2-C_1+(P+B_3)\theta$	$C_0-K_2-A-\theta V$
(0,1,0)	B_1-F	$C_1-C_2-\theta(P+B_3)$	$C_0-K_2-A-\theta V$
(1,0,1)	$F-B_1-(P+B_2)(1-\theta)$	$C_2-C_1+P-B_3(1-2\theta)$	C_0-A-V

In this paper, we focus on the asymptotic stability point of community group buying platforms that choose the "Positive supervision" strategy, i.e. $(1,0,0),(1,1,0),(1,1,1),(1,0,1)$.

Since in the eigenvalues of $(1,1,0)$ have $F>0$, $(1,1,0)$ are not likely to be the evolutionary stability point.

The equilibrium point $(1,0,0)$ is the evolutionary stability point when the conditions $F-P(1-\theta)<0$, $\theta B_3-C_1+C_2+P<0$, $A-C_0<0$ are satisfied. In this case, because the income of the suppliers' fraudulent transactions is greater than the loss, so the suppliers will choose the strategy of "Fraudulent transactions" for their own interests,thus rendering the supervision of the community group-buying platforms useless.

The equilibrium point $(1,0,1)$ is the evolutionary stability point when the conditions $F-B_1-(B_2+P)(1-\theta)<0$,

$-[C_1-C_2-P+B_3(1-2\theta)]<0, C_0-A-V<0$

are satisfied. The process of analysis is similar to $(1,0,0)$ and will not be repeated.

The equilibrium point $(1,1,1)$ is the evolutionary stability point when the conditions $F-B_1<0$, $C_1-C_2-P+B_3(1-2\theta)<0, C_0-A<0$ are satisfied. Community group-buying platforms carry out positive supervision, suppliers conduct honest transactions, consumers protect their rights, tripartite collaboration, to promote the healthy development of the platform.

4. Simulation analysis

The ultimate goal of this study is to achieve the ideal state of positive supervision of community group-buying platforms, honest transactions of suppliers and consumers rights protection, and to find the corresponding strategy that tends to the asymptotic stability point. Based on the above evolutionary game model, numerical simulation analysis is carried out according to the initial conditions of the equilibrium point $(1,1,1)$.

The corresponding parameters are set as

$$B_1=5, B_2=6, B_3=3, F=1, P=6, \theta=0.6, R=8,$$
$$C_0=4, C_1=6, C_2=4, K_2=2, A=6, V=12$$

Let x_0, y_0 and z_0 be the initial probabilities of participation of community group-buying platforms, suppliers and consumers respectively. Let the initial state be $(0.5,0.5,0.5)$. The simulation results obtained are shown in Figure 1.It can be seen that, with the increase of time under the constraint, the stable point eventually reached $x=1, y=1, z=1$ by the three-party evolutionary subject, i.e.the ideal state of positive supervision of the community group-buying platforms, honest transaction of suppliers and consumers rights protection, thus verifying the validity of the above conclusion.

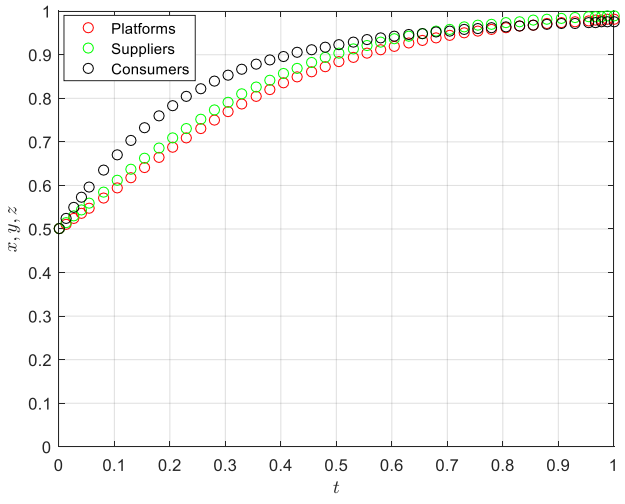


Figure 1 Three-party evolutionary stabilization strategy

Next, we analyze the influence of changes in different parameters on the rate of suppliers choosing "Honest transactions" under the condition of community group-buying platforms positively supervision and consumers rights protection.

(1) With other parameters stable, the evolutionary path of suppliers strategy when θ is taken as $0.2,0.5,0.8$ respectively is shown in Figure 2. It can be seen that the larger the θ, the more likely the suppliers tend to 1 and its rate is accelerating.This indicates that the greater the discovery rate of community group-buying

platforms under negative supervision, the more likely suppliers are to choose "Honest transactions".

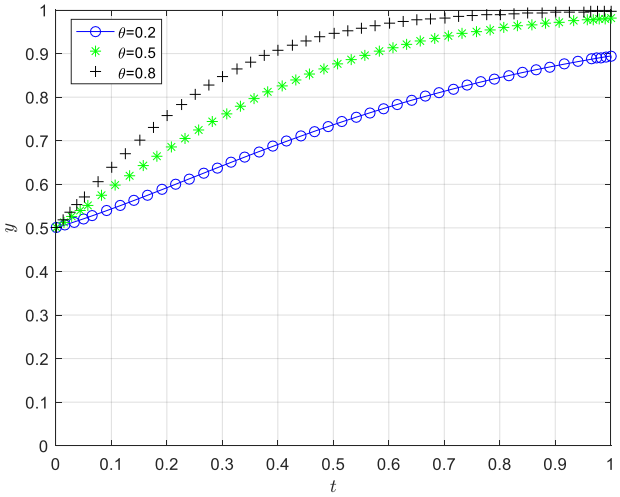
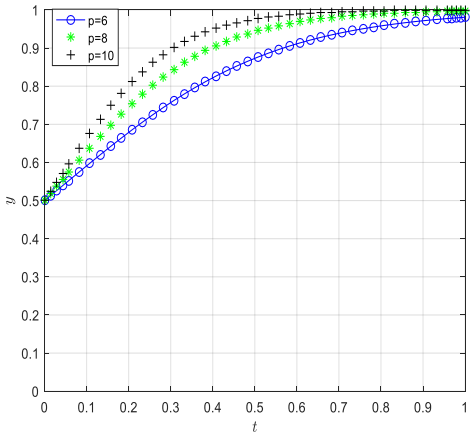


Figure 2 The effect of θ on the rate of supplier choice honest transaction

(2) With the other parameters stable, the evolutionary path of the suppliers strategy when P is taken as $6, 8$ and 10 respectively is shown in Figure 3. It can be seen that as P gradually increases, the rate at which suppliers tend to move towards 1 gradually accelerates. From Figure 4, it can be seen that B_3 has the same effect on suppliers behaviour as P, so no further analysis will be carried out on.



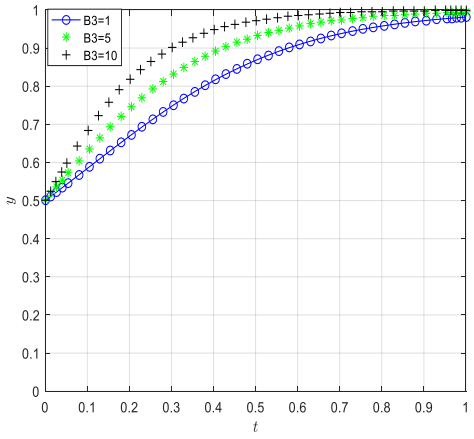


Figure 3 The effect of P on the rate of supplier choice honest transaction **Figure 4 The effect of B_3 on the rate of supplier choice honest transaction**

(3) With the other parameters stable,the evolutionary path of the suppliers' strategy when ΔC is taken as $1.5, 2.5, 3.5$ respectively is shown in Figure 5. It can be seen that as ΔC gradually decreases, the rate at which suppliers converge to 1 gradually accelerates.

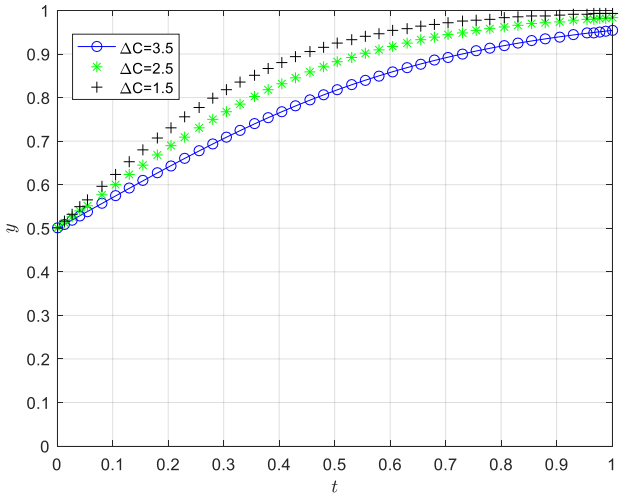


Figure 5 The effect of ΔC on the rate of supplier choice honest transaction

5. Conclusion

In this paper, an evolutionary game model of the community group buying platforms, suppliers and consumers was constructed and simulated for the equilibrium point (1,1,1). Simulation analysis shows that in the case of positive supervision of the platforms and consumers rights protection, the rate at which suppliers tend to engage in honest trading is related to the punishment of the platforms for suppliers' fraudulent trading behavior, the past reputation of suppliers, and the difference between the cost of suppliers' honest trading and the cost of fraudulent trading.

The healthy and orderly and prosperous development of community group buying platforms requires the joint efforts of all parties involved. The following recommendations are made to this end.

(1) The platforms should refine and clarify the suppliers trading system; and increase the penalties for suppliers when they are found to have traded fraudulently.

(2) The platforms should reward suppliers with a good reputation based on positive consumers reviews, thereby reducing the difference in cost between honest and fraudulent suppliers transactions.

(3) The platforms should simplify the after-sales process and create convenient conditions for consumers to protect their rights, thus giving full play to the power of consumers supervision and making up for the lack of efficiency of platforms supervision.

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